The wildlands connectivity dilemma: a graph-theory computational approach

Simon Jean*[1] and Lauriane Mouysset[1]

[1]CIRED, Ecole des Ponts, AgroParisTech, EHESS, CIRAD, CNRS, Université Paris-Saclay, Nogent-sur-Marne, France

January 25, 2024

Abstract

Background: Fuel treatment operations help to mitigate the spread and severity of wildfires in numerous ecosystems. As they aim at fragmenting the fire landscape, they also fragment wildlife habitat. This poses a dilemma for land managers, in the form of a trade-off between lowering wildfire patch connectivity and maintaining wildlife habitat connectivity. Previous studies have investigated the spatial allocation of fuel treatments over time, mostly without specific care devoted to biodiversity, in a variety of case studies. However, they lack generality and an interpretative framework. We use dynamic programming and graph theory on every possible theoretical landscape configuration to gain a general understanding of the allocation of treatments over space and time and the corresponding landscape properties with various habitat connectivity targets.

Results: Our results show that all initial landscapes converge to steady-state landscape cycles. Moreover, we show that there exist optimal trajectories that significantly reduce wildfire risk while safeguarding habitat connectivity. As the policy budget increases, more risk reduction is achieved, albeit with a decreasing marginal efficiency, and more steady-state cycles emerge. As habitat targets increase, increasing the budget is of no effect, and risk increases, while the number of steady-state cycles decreases. Landscapes are less risky, more fragmented, and diverse when the budget is large and biodiversity targets are low, while they are more compact and less diverse when the opposite is true. Treatment allocation follows graph centrality measures, and central cells are treated first. When the budget increases, fewer central cells (i.e. edge patches) are treated as well. When biodiversity targets increase, central cells are no longer treated as they decrease habitat connectivity. Treatment is reshuffled to the edges of the landscape.

Conclusion: Computational experiments generalize existing results. Using graph theory, general insights can be gained, and help managers faced with multiple objectives in forested landscapes. From a policy perspective, in the face of climate change, increasing treatment budgets should be a priority to avoid increasing damages. A key guideline is treating a variety of seral stages to create landscape diversity, mitigate risk and guarantee the connectivity of wildlife habitat.

Keywords : Fuel treatment, connectivity, wildfire risk, wildlife habitat, spatial optimization, graph theory

1 Introduction

Hazardous and intense wildfires threaten forest resilience and can cause ecosystem shifts (Coop et al. 2020). They also cause dramatic impacts on biodiversity across taxa (Wintle et al. 2020). Moreover, intense wildfires cause human damages, in the form of direct asset losses: in 2018, wildfires in California have caused $ 27 billion (Wang et al. 2021). Indirect costs are also of concern, especially related to wildfire smoke (increase in PM 2.5 concentrations have important health impacts (Burke et al. 2023, Heft-Neal et al. 2023), recreation values are affected in the US, amounting to $USD 2.3 billion (Gellman et al. 2023)). Eventually, large wildfires are of importance in the face

*Corresponding author : jean@centre-cired.fr

of climate change releasing a lot of greenhouse gas and reducing the atmospheric carbon sinks (Zheng et al. 2023, Sweeney et al. 2023). Global warming affects water supply and fuel moisture (Jolly et al. 2015, Abatzoglou and Williams 2016, Ruffault et al. 2018), and is projected to increase the frequency, severity, and magnitude of wildfires (Wasserman and Mueller 2023). Recent wildfire events in California (since 2018), in Australia (2019-2020), and in Europe (France, Portugal, Greece in 2022) have epitomized these trends.

In numerous regions, such as conifer forests in California (Vaillant et al. 2009, Kalies and Yocom Kent 2016, Low et al. 2023), eucalypt forests in South Western Australia (Burrows and McCaw 2013, Boer et al. 2009, Florec et al. 2020), southern Europe (Fernandes et al. 2013), evidence shows that fuel treatments (e.g. prescribed burns, mechanical thinning and managed wildfires), can mitigate wildfire intensity and spread. Land management agencies have historically implemented these policies in Australia (Burrows and McCaw 2013), Europe, and the United States (and are projected to ramp up, for example under the Infrastructure Investment and Jobs Act of 2021 in the US). Understanding the spatial allocation of treatments, as climate change impacts negatively both costs and feasibility, is a major driver of policy success (Williams et al. 2017, Florec et al. 2020).

By changing the structure of the landscape, fuel management operations also affect the structure of biodiversity habitat, notably, its structural connectivity (Taylor et al. 1993). Maintaining habitat connectivity, through wildlife corridors, landscape links, and ecoducts (Turner 2005, Turner and Gardner 2015), is instrumental in mitigating the biodiversity crisis. Species richness and diversity are intimately linked to landscape connectivity (Olds et al. 2012, Tian et al. 2017, Velázquez et al. 2019) and are necessary to maintain ecosystems in the future. The impact of fuel treatments on biodiversity remains a debated topic. Evidence suggests that maintaining a variety of vegetation types and ages on a patchy landscape maintains a 'fire mosaic' (Sitters et al. 2015) (e.g. landscape level variations in habitat types that provide habitat to an ecological community) or that fuel treatment can be beneficial to wildlife (Saab et al. 2022, Loeb and Blakey 2021) and even restore local populations (Templeton et al. 2011). On the other hand, treating at too high a frequency may be detrimental to biodiversity (Bradshaw et al. 2018). Overall, implementing fuel treatment challenges the connectivity of wildlife habitat. In this context, understanding the trade-offs between risk reduction and biodiversity conservation, as well as the spatial patterns of operations that could reconcile the two objectives is key. In this study, we investigate the spatial allocation of fuel treatments to optimally reduce wildfire risks while maintaining biodiversity habitat.

A substantial literature has applied optimization techniques to tackle the spatial allocation of fuel treatments. Analytical (Finney 2001), simulation-based (Finney 2007, Rytwinski and Crowe 2010) or mixed-integer programming techniques (Wei et al. 2008) have solved the allocation of treatments in a static framework. Given the dynamic nature of fuel growth, studies based on mixed-integer dynamic programming (Wei et al. 2008, Minas et al. 2014, Rachmawati et al. 2015; 2016) have studied the temporal and spatial allocation of fuel treatments on real and simulated landscapes. While they solve the spatial treatment allocation problem in forests, these articles fail to acknowledge the multiple uses and objectives land planners have to consider, such as habitat conservation. Several articles have devoted their attention to the spatial allocation of treatments while conserving habitat, and investigated the trade-offs between risk reduction and biodiversity conservation, using spatial heuristics (Calkin et al. 2005, Lehmkuhl et al. 2007) and linear programming (Williams et al. 2017, Rachmawati et al. 2018). Most of the existing literature focuses on case studies and lacks a general interpretative framework to generalize its results. Graph theory offers a toolbox suited to analyze the properties of connected patches of land with varying characteristics, and has extensively been applied in landscape ecology (Urban and Keitt 2001, Minor and Urban 2008, Rayfield et al. 2016). Recent research focusing on the allocation of fuel treatments has leveraged tools from graph theory (Matsypura et al. 2018, Pais et al. 2021a). Reconciling habitat and wildfire risk mitigation using graph theory is a recent research endeavor (Rachmawati et al. 2018, Yemshanov et al. 2022) and has focused on specific case studies.

In this article, we leverage graph theory on an exhaustive set of theoretical landscapes to study the general patterns of treatment allocation emerging from a multi-objective, dynamic, and integer landscape management problem, governed by connectivity. We analyze all the landscape configurations resulting from a 20-period planning horizon, for regular grid landscapes, in a graph theoretical perspective. In doing so, we examine the fuel treatment patterns resulting from all the range of habitat connectivity, in order to characterize long-term landscape properties. We characterize the landscapes using a range of ecological indicators and find general mechanisms and guiding principles applicable to a broad class of settings, to guide decision-makers and foster new efficient multi-objective graph theory algorithms.

Our contributions are several. First, we provide a spatial framework to understand the trade-offs between wildfire risk reduction and biodiversity conservation. Using graph theory, we derive general principles regarding the spatial characteristics of landscapes and treatments from an exhaustive set of theoretical landscapes to guide policymakers as well as future research in heuristics to reconcile conflicting land-based phenomenons. Eventually, we characterize the risk and biodiversity profiles consistent with a changing climate, where windows of opportunity are shorter and costs of treatment larger, and the associated spatialized treatments.

2 Methods

2.1 Theoretical model

We consider theoretical landscapes represented by a regular grid of $n \times n$ cells with a forest seral stage succession module. We use a stylized representation of the link between vegetation age, habitat, and wildfire risk. We denote by A_t the set of equal, standardized area cells in the theoretical landscape of dimension $n \times n$ (hereafter referred to as being of size$= n$) in period t. Each cell a_i at time t is characterized by a seral stage: absent, young, or old. At each time step, it changes stage until it is in the 'old' stage, where it remains. Upon treatment, a cell's seral stage is set to 'absent' (see equation A.1 in appendix A).

A cell offers wildlife habitat once it is 'mature' (eg seral stage is at least 'young'), i.e, when the time elapsed since the last burn reaches the maturity threshold (eq. A.2). We assume that habitat quality is uniformly distributed among habitat patches and that neighboring cells are reachable, conditional on being 'mature'. After the wildlife habitat maturity threshold, a cell can turn at critical risk of wildfire during a 'normal' hot season. We assume an Olsen-type model of flammability (Olson 1963, McCarthy et al. 2001), where age is the main predictor. Therefore, after the 'high fuel load' threshold is crossed, the cell is regarded as 'high risk' from then on, until treatment suppresses this risk (eq. A.3).

We define cells to be connected if (i) they are within an 8-cell neighborhood and (ii) share the same status. Regarding biodiversity, we focus on general characteristics related to landscape structural connectivity rather than functional connectivity, as we are agnostic about effective species (Fahrig et al. 2011). We assume that species are able to disperse from one patch to another, and that habitat quality is uniformly distributed conditional on habitat being available. We consider the wildfire risk through the lens of potential spread, which is only driven by fuel. Consistent with the literature (see Peterson et al. (2009), Pais et al. (2021b), Gonzalez-Olabarria et al. (2023)), a wildfire can spread in any direction, conditional on neighbor cells with high risk. However, if surrounding cells do not display high risk, fire does not spread.

We use a network structure to apprehend the landscapes. We transform A_t the set of cells constituting the landscape into graphs G_t whose vertices V_t (or nodes) are the cells in the landscape, and edges E_t represent the connections between cells. We partition the landscape in two graphs, G_{B_t} and G_{F_t}, each describing the network of mature habitat and risky patches (see fig. 1 for a representation). Landscape ecology has long used numerous, theoretically grounded indicators to analyze landscapes (Urban and Keitt 2001, Minor and Urban 2008). We use a global connectivity indicator that satisfies Pascual-Hortal and Saura (2006) criteria, grounded in graph theory, that offer a reformulation of Rachmawati et al. (2016) (see Appendix A.3).

We define the global connectivity index of habitat and risky patches in landscape $A(t)$ as:

$$H_i(A(t)) = card(V_{i_t}) + 2 \times card(E_{i_t}) \text{ with } i \in \{B, F\} \tag{2.1}$$

This indicator considers that a habitat patch is connected to itself (i.e, within a habitat patch, there is no barrier) and whether it is connected to other patches. It implies lower connectivity when the distance between patches increases, attains its maximum value when a single habitat patch covers the whole landscape, indicates lower connectivity as the habitat is progressively more fragmented, considers negative the loss of a connected or isolated patch, and detects as more important the loss of bigger patches, of key and less important steppingstone patches.

To manage the expected damages resulting from wildfires, the land planner can decide to undertake specific treatments, in the form of a combination of controlled burns and/or mechanical thinnings. Upon treatment, we assume that vegetation age in the cell is reset to 'absent': the wildfire risk vanishes, but so does the habitat and its connection to surrounding cells. Given the tension between maintaining habitat and reducing wildfire risk, the land planner aims to minimize a deterministic measure of connectivity of the high fuel loads in the landscape while maintaining a given level of biodiversity habitat connectivity under a budget constraint, over a planning horizon of length T. For the sake of the analysis, we focus on two layers of complexity over time and space: risk connectivity and biodiversity habitat. We do not consider heterogeneity in the economic costs or benefits (i.e, homogeneous treatment costs and no patch-specific asset to protect). The framework is however amenable to such a prioritization. We also assume that the budget cannot be banked, and has to be utilized in each period, consistent with operational rules. Moreover, as the budget is constrained in each period, the measure of risk is bounded and the planning horizon is finite, we rule out discounting and assume each generation matters as much to the social planner.

The optimization problem is :

$$\min_{x} \left[\sum_{t=1}^{T} H_F(A(t)) \right] \tag{2.2}$$

Such that:

$$A_i(t+1) = \min((A_i(t)+1)(1-x_i(t)), 2),\ t = 1, ..., T,\ \forall i \in C \tag{2.3}$$

$$H_B(A(t)) \geq Biod,\ t = 1, ..., T \tag{2.4}$$

$$\sum_i x_i(t) \leq Budget,\ t = 1, ..., T \tag{2.5}$$

$$A(0) \text{ given} \tag{2.6}$$

$$x(t) \in \{0,1\}^{n^2} \tag{2.7}$$

We abstract from decision-making in a risky environment, as it has been extensively described in economics and decision theory (Mouysset et al. 2013). Moreover, we mimic the role of risk aversion by varying the level of habitat connectivity constraint the decision maker chooses. We solve the dynamic, integer program of the landscape manager using dynamic programming. Dynamic programming provides a temporal decomposition of the initial problem defined over T periods, into T simpler problems, as it relies on the 'optimality principle'[1]. Second, it provides feedback controls which are know to be more adaptive especially if shocks occur or uncertainties affect the states or the dynamics of the system . The outputs of the method are both the optimal policies $x_j^*(t, A)$, i.e, the sequence of optimal controlled burns, and the optimal states $A_j^*(t, A_0)$ resulting from the optimal policies and the initial conditions

[1] "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision". (See Bellman (1957), Chap. III.3., p.83)"

We solve the land planner's problem for every possible initial condition, thus giving rise to general conclusions on the properties of landscapes and treatments emerging from this problem, under various budget scenarios to account for climate change.

2.2 Lanscape indicators

To characterize the managed landscapes, we mobilize several indicators from landscape ecology and graph theory (see appendix B). First, we account for the risky and habitat areas in the landscape (eq. B.1). Second, to assess landscape connectivity/fragmentation and diversity in the context of fire mosaics (Bradstock et al. 2005), we use our connectivity metric (eq. 2.1), the number of components e.g. the number of maximal connected subgraphs within the graph, that is not connected to other vertices (eq. B.2) for the risky cells graph, as well as the corresponding areas. To specifically assess landscape diversity, we use the Simpson index (Simpson 1949) on seral stages (eq. B.3)[2]. However, the Simpson index does not account for the diversity of spatial patterns: a checkered landscape with two seral stages would be as diverse as a landscape with two large patches for each seral stage, according to the Simpson index. Therefore, we use the landscape shape index (eq. B.4), a normalized ratio between the perimeter of biodiversity habitat and its area (Patton 1975, McGarigal and Marks 1995). To disentangle the correlated effects of perimeter and area that affect the landscape shape index, we use a land type heterogeneity index, that averages the probability that, for each cell, neighbors in the 4 cardinal directions share the same land types (eq. B). The index ranges between 0, when the land type is the same across the whole landscape, to 1, in a checkered landscape. The index assesses whether the landscape is a mosaic (Bradstock et al. 2005), and if it displays structural diversity, conducive to diverse communities and functional diversity.

2.3 Computational experiments

Our problem can be viewed as a critical node detection problem, i.e, a problem of locating the nodes that best degrade connectivity metrics (Arulselvan et al. 2009). Problems of the critical node class are computationally difficult (e.g. NP - Hard) in a single graph (Arulselvan et al. 2009, Matsypura et al. 2018). Efficient heuristics to find near-optimal solutions exist and leverage perturbations around local solutions (Arulselvan et al. 2009, Zhou and Hao 2017). Our problem is a constrained, integer optimization problem that constrains not only the set of nodes to be removed but also metrics relative to a larger graph structure (e.g. supergraph of risky patches), biodiversity habitat. For this reason, existing heuristics may not perform well on our problem. Moreover, the complexity of our combinatorial problem increases with landscape size and vegetation age class exponentially, displaying the 'curse of dimensionality' (Bellman 1957). Therefore, we limit ourselves to studying all the initial conditions in landscapes of size $n = 3$ and 4. While this formulation appears simplifying, it encapsulates the main mechanisms displayed

[2]Similar results can be found with the Shannon index (Shannon 1948). To avoid issues related to degenerate values and logarithms, we focus on the Simpson index.

in similar models (Rachmawati et al. 2016; 2018). It allows us to solve the problem for the whole set of initial conditions, for the whole range of biodiversity habitat connectivity constraint values, over 20 years. In our analysis, we consider a range of budget values for treatment costs normalized to 1. As common in the literature, we can express the budget as a share of land being treated ranging from 5% to 44% of the surface area. These values encompass historical and projected policies in Australia (Burrows and McCaw 2013), the United States (Office 2019) and Southern Europe (Fernandes et al. 2013).

Of all the 3^{n^2} initial conditions landscapes, we only keep landscapes that are unique up to a permutation[3]. This results in a sharp reduction of landscapes to consider, from $19,683$ initial conditions to 2861 unique initial landscapes for $n = 3$, and from $43,046,721$ initial to $5,398,082$ unique initial landscapes for $n = 4$. We focus on exact optimal solutions for all the initial conditions of these small-scale landscapes and implement our own solution algorithm in Python 3.9.13. Data and code are publicly available.

3 Results

3.1 Steady states

Our simulations show that 100% of the initial landscapes converge in finite time towards a steady state solution, that minimizes wildfire risk while satisfying budgetary and habitat connectivity requirements. Steady states are landscape cycles with finite periods. Analyzing the steady-state cycles (and the unique landscapes that form them) drastically reduces the set of landscapes to analyze: they represent 2% (resp. 0.001%) of the initial landscapes of size $n = 3$ (resp. $n = 4$). Our model highlights the convergence of landscapes towards types that can be managed to deliver several objectives. As landscape size increases, the number of steady state landscape cycles increases, but the power of convergence increases as well (e.g. ratio between initial configurations and effective steady state landscapes): from 19 683 initial landscapes when $n = 3$, 51 steady states emerge and from 43 046 721 initial landscapes when $n = 4$, at most 95 diverse steady-state landscapes emerge. Focusing on steady states makes all the more sense as landscape size increases.

Eventually, figure 2 shows that conditional on data availability on every patch, the more the decision maker wants to conserve biodiversity, the fewer steady-state landscapes she has to consider. An increase in the habitat requirement reduces the room for maneuver. Indeed, budget acts as a complexifying factor: the larger the budget (relative to costs), the larger the set of steady-states to consider. Aiming for relatively large habitat connectivity reduces the set of viable strategies to be considered and can more efficiently guide policy.

[3] That is to say, landscape A is included in the set of initial conditions $\mathcal{I}$ if and only if for any element B in $\mathcal{I}$, A is not a permutation (eg can be obtained through rotations or symmetries) of B

3.2 Wildfire risk reduction and habitat connectivity in steady state landscapes

Figure 3 shows the wildfire risk reductions and habitat requirements normalized by their respective maximum values for landscapes of size $n = 3$ and 4. The maximum value for both risk and habitat corresponds to a landscape covered in 'old' vegetation, which we take to be the counterfactual. Randomly assigned treatments do generate risk reductions but are not cost nor habitat-efficient. Following our spatial optimization procedure, it is clear that implementing fuel treatment reduces wildfire risk while supporting biodiversity habitat. Figure 3 shows that these two objectives come as a trade-off, albeit moderate: indeed, increasing habitat requirements increases the remaining risk, but there are combinations that can satisfy large habitat connectivity and risk reductions. Budget is a key factor in risk reduction, as it relaxes the trade-off between the two objectives: increasing the budget reduces the wildfire risk while maintaining a range of biodiversity constraints. When habitat constraints are large, however, the marginal effect of budget is limited, and a larger remaining risk needs to be accepted. For example, with a budget of 25% of land to be treated (with landscape size $n = 4$), and no habitat constraint, risk can be reduced up to 80% compared to the counterfactual scenario. However, when the habitat constraint is at 60%, only 70% of risk reduction can be achieved. Moreover, this risk reduction can be achieved with a lower budget. Conversely, as the costs of treatment increase, for a stable budget, the remaining risk increases sharply, and factoring in habitat requirements in the decision-making is not necessary for targets below 80%.

3.3 Properties of steady state landscapes: surface, fragmentation, and diversity

Figure 4 displays, for each class, the most frequent steady-state cycle for landscapes of size 3 and 4 for each biodiversity target. Figure 5 shows the indicators relative to the surface and components of the high-risk graph and figure 6 shows the indicators related to diversity, both for landscapes of size $n = 3$ and 4, averaged over all the steady-state landscape cycles.

Previous results show that budget increases risk reduction, conditional on habitat connectivity constraint being low. Focusing on zones A and A' of the panels of figure 5 shows that risk reduction primarily comes from a reduced surface (panels 5a and 5b), and an increase in the number of components, i.e, disconnected high-risk patches (panels 5e and 5f). Overall, the high-risk area is reduced and the number of components increases, thus resulting in smaller largest high-risk component area (panels e and f). As more connected habitat area needs to be protected, the high-risk surface increases (fig. 4 panels 5a and 5b) and the number of high-risk components drastically reduces. The landscapes collapse to the same dominant structure (fig. 4), where the high-risk area is (almost) maximal and there is one large, well-connected component. Overall, landscapes are riskier but also feature larger, better-connected biodiversity habitat. For large budgets (e.g. 3 and 4), these effects are non-trivial: the number of components (weakly) increases first, small components either disappear or increase in size (see figure 4 for budget 4 in panels A', B' and C'), risky patches are reallocated to connect separated components before the high-risk surface increases.

Landscape diversity unambiguously increases with the budget (panels 6a,6b, sections A and A'). As more units are treated, the evenness of seral stages increases in the landscapes. When the habitat objective is low, the spatial diversity of landscapes increases with the budget (panels 6c, 6d): even though the relative area of habitat decreases with the budget, the shape of habitat is more irregular, and the landscape is more of a mosaic. In this context, cells with a 'young' seral stage act as stepping stones and corridors between high-risk habitat patches. When habitat objectives increase, diversity collapses both quantitatively and qualitatively (fig. 6). The Simpson index collapses from panels A (resp. A') to G (resp. F'), as land types gradually homogenize (see fig. 4 for an illustration) across all budgets. Moreover, landscapes form less of a mosaic, and are more clumpy, as displayed by the LSI and Land type heterogeneity index. Overall, for large habitat targets, landscapes tend to homogenize and to be better connected, although less quantitatively and qualitatively diverse.

Results are consistent across landscape sizes while they display more variability for size $n = 3$, as border effects play a larger role.

3.4 Spatial allocation of optimal management at the steady-state landscape cycle

Figures 7a and 7b display the number of fuel treatments in the steady-state cycles, for various budgets and habitat connectivity constraints. Treatment allocation follows the evolution of the high-risk area (fig 5a and 5b): the larger the budget, the larger the treated area, the budget constraint is always satiated. However, when biodiversity targets increase, the budget constraint is no longer satiated.

Figures 7c and 7d display the average spatial location of treatments in the steady state cycles. The darker the cell, the higher the frequency of treatment. First, not all cells are equally treated. For low levels of biodiversity constraint, panels A and A' of figures 7c and 7d show that central cells are primarily treated, and when the budget increases, cells on the edges get treated, while corner cells are never treated. In the context of critical node detection, when the ecological requirements are low, the high-risk graph is primarily considered, and nodes with the most cost-efficient risk reduction, i.e, with the largest degree are targeted. Once the most connected cells are treated, lower-degree cells get treated.

When habitat constraints increase, several effects come at play. Not only does the number of treatments decrease, but the spatial allocation also changes. For example, in panels A and B for budgets 3 and 4, panels C and D for budget 2 and panels E and F for budget 1 in figure 7c, the number of treatment remains the same but is spatially reallocated to lower degree nodes. Treatments are spatially reallocated before being reduced. In this context, as the relative weight of the habitat graph increases, treating the most cost-efficient risk-reducing nodes also degrades habitat connectivity. Therefore, as habitat targets increase, edge and corner (e.g. low degree nodes) are being treated and habitat connectivity is maintained.

4 Discussion

4.1 Confirmation and generalization of existing results

Our analysis of the exhaustive set of initial conditions for small-scale landscapes confirms existing results in the literature. We argue that they bring robust evidence and complement the existing literature to derive general conclusions.

Our model encompasses 3 seral stages and 1 composite vegetation type and proves the convergence of every initial condition to a steady state cycle, irrespective of the initial configuration. We extend Minas et al. (2014) that find convergence patterns for *homogeneous* landscapes only, i.e, landscapes where the initial vegetation age is uniformly distributed. We show that in the event of environmental perturbations that do not disrupt ecosystem dynamics, an appropriate policy can recover the previous equilibrium risk and habitat. We hypothesize that as long as the risk/ seral-stage relationship reaches a plateau for every vegetation type on the landscape, convergence should be observed.

Our results display a concave production possibility frontier (PPF) between wildfire risk reduction and habitat connectivity, consistent with PFF literature (Arthaud and Rose 1996, Calkin et al. 2005). Our results also confirm that trading one objective for the other is not as efficient as increasing the policy budget to reconcile objectives. We show that increasing the policy budget nonetheless has diminishing returns for risk reduction, as highlighted by Wei et al. (2008), Yemshanov et al. (2021) and Pais et al. (2021b).

Our study yields clear results in terms of landscape ecology, leveraging concepts from landscape ecology, and highlighting the spatial mechanisms underlying the shape of PPF. We show that treatment allocation targets the most central nodes first and then focuses on less connected nodes (e.g cells closer to the border of the landscape) when habitat goals are low. In doing so, we do find general treatment allocation principles where previous studies on larger landscapes could not (Minas et al. 2014, Rachmawati et al. 2016), generalize smaller scale (Konoshima et al. 2008) and case study specific (Yemshanov et al. 2021, Pais et al. 2021a) results.

Leveraging a dynamic integer programming, graph theoretic framework on small-scale landscapes, we show that cell-level metrics help formalize and understand the drivers of treatment allocation and rationalize existing results. Furthermore, we show that while prioritization approaches based on a graph theoretic framing fare very well in an unrestricted set-up, including biodiversity habitat targets augments the problem's complexity. We generalize case studies (Yemshanov et al. 2022) and show less central high-risk nodes need to be targeted to achieve risk reduction and safeguard biodiversity habitat.

4.2 Caveats and methodological perspectives

Our analysis tackles the exhaustive set of landscapes of size $n = 3$ and 4. Our approach allows us to study the steady-state patterns emerging from any initial condition, replicates existing results in larger landscapes, and sheds light on the mechanisms underlying the wildland dilemma. Increasing landscape size is incompatible with this approach, as we would run into a dimensionality curse (Bellman 1957). To conserve our exhaustive approach, different proof mechanisms would be required. Nonetheless, if landscape size is of the essence for actual policy recommendation, so are other layers of information such as habitat quality, treatment costs, and values at risk heterogeneity. These other layers would reduce the computational burden, and we believe our results, targeting the most cost-efficient, risk-reducing, and habitat-conserving strategies, would still apply.

In our model, we use a simple relationship to characterize the link between the seral stage, habitat formation for a single species, and wildfire risk and severity. This choice is motivated by the existence of a lower bound for a fire return interval and drives our ability to adopt our exhaustive approach. Increasing the number of seral stages would help to complexify the relationships governing habitat formation and wildfire risk and severity: in some ecosystems, wildfire risk and severity may be higher for young vegetation than for older and may not be linear (Taylor et al. 2014). On the other hand, some species may require old-growth forests to survive, not 'young' forests, and old-growth forests may also be more fire-resilient (Lesmeister et al. 2021). As the number of seral stage augments, convergence towards steady-state landscape cycles would take longer, but we hypothesize it would still occur. Moreover, as long as wildfire risk and habitat quality are in conflict, a trade-off would govern treatment allocation. Multiple seral stages may be targeted for fuel treatment, depending on their location and properties, but we claim the general mechanism would still apply: in a graph weighted for different risk and habitat properties, centrality and connectivity would still guide treatment allocation.

We implicitly assume that focusing on a given species' habitat would also provide habitat for a variety of species and be conducive to functional diversity. However, this does not imply that all species would benefit from maintaining a given habitat type (Saab et al. 2022). Moreover, the lack of structural diversity may cause the trophic web of the targeted species to collapse. Therefore, management objectives should include structural diversity. In this case, landscapes could not satisfy extreme habitat connectivity targets and diversity targets. For intermediate goals, however, we claim that treatment allocation would still aim at fragmenting the landscape, and node centrality and connectivity would still govern allocation.

Eventually, we chose to abstract from a stochastic ignition process affecting the landscape. As a thought experiment, imagine a Bernoulli-distributed, high-risk area independent probability of ignition in each period. If part of the landscape ignites, all that remains is the unburnt habitat, while if not, all habitat remains. A decision-maker faced with maximizing the expected payoff in this scenario would solve the reciprocal of our problem. On the one hand, she has to ensure that the high-risk cells in the landscape are not 'too' connected, to maximize the remaining habitat in the event of a wildfire. On the other hand, she wants to maximize connectivity for wildlife when there is no wildfire. As a result, the trade-off she faces, and the resulting spatial allocation of treatment would be the same. The stochastic nature of ignition may change the steady state cycles, but convergence would not be impossible. If the probability of wildfire increases, she focuses more on maintaining a 'young' seral stage over the landscape. In this setting, increasing the probability of ignition would act as a decrease in our habitat target as well as an increase in the budget available for policy. With our model, we are able to disentangle these two effects and understand how each constraint would play. We claim we match with actual policy, where the budget is not fully endogenously determined.

4.3 Conclusion and policy relevance

While there is a *dilemma* for land managers between lowering wildfire risk and severity and maintaining species habitat connectivity, reconciling the two objectives is not a dead end. This is an important result for land planners as biodiversity habitat targets are gradually included in policy agendas (for example, the recent pledge by the participants to the Conference of Parties on Biodiversity in Montreal to preserve 30% of land and oceans by 2030 for biodiversity[4]). It shows that if policymakers can commit to a given budget over time, these biodiversity targets can be reached and a management cycle that minimizes wildfire risk can be implemented in wildlands. Moreover, as steady-state cycles are reached, the uncertainty over future land uses is resolved while achieving policy goals.

In the face of climate change, treatment costs are expected to increase (Kupfer et al. 2020). The decreasing marginal efficiency of budget to reduce risk highlights that as climate change increases the costs of treatments, risk, and damages will increase at an increasing rate, unless the budget is changed accordingly.

Our analysis shows that budget should be determined by factoring a careful, *ex-ante* analysis of treatment costs, the policy maker's risk aversion towards a measure of wildfire risk and severity, and ecological preferences. Indeed, low budget-to-cost ratios are incompatible with high risk and severity aversions and/or large ecological requirements.

As wildfires and biodiversity habitat destruction are challenges in the face of global warming, finding policy guidance tools is of the essence. Many studies focus on specific case studies or limited ranges of potential initial conditions. We develop a simplified ecological model of habitat and wildfire connectivity to guide policymakers in the form of general principles. Reducing wildfire risk and accommodating wildlife habitat is possible with carefully designed policies, where budget plays a key role. However, it is impossible to achieve drastic risk reduction without harming biodiversity habitat. General principles of treatment allocation in the landscape are derived, and the concepts of graph theory provide an operational toolbox to understand the underlying mechanisms. Landscape patches that display high wildfire risk seral stages and are well connected to other patches should be treated first. When habitat targets are included, tackling lower-risk patches is of the essence to maintain habitat connectivity.

Our article summarizes and generalizes how policies should be implemented, both in terms of budgets and spatial allocation, to protect and enhance ecosystem health.

[4]See Target 2 in the Keunming-Montreal Global Diversity Framework, 2022

5 Declaration

5.1 Acknowledgments

This research was conducted while SJ was on leave at the Environmental Markets Lab, UC Santa Barbara. We acknowledge support from the Center for Scientific Computing from the CNSI, MRL: an NSF MRSEC (DMR-1720256) and NSF CNS- 1725797, at UC Santa Barbara. Moreover, the authors are grateful to the editor and X anonymous referees, as well as participants to the Columbia Interdisciplinary PhD Workshop in Sustainable Development and the BINGO group at CIRED for their valuable comments.

5.2 Data availability

Given its size, steady-state cycle data is available upon request from the authors. Code for replication is available at https://github.com/sim-jean/Landscape_connectivity_dilemma

5.3 Author affiliation

CIRED, Ecole des Ponts, AgroParisTech, EHESS, CIRAD, CNRS, Université Paris-Saclay, Nogent-sur-Marne, France

5.4 Competing interests

The authors declare no conflict of interest.

5.5 Contribution

LM designed the study, SJ ran the computational experiment, SJ and LM analyzed the results and wrote the manuscript.

Appendix

A Theoretical model

A.1 Vegetation dynamics

In cell i at time t, vegetation ages $A_i(t)$ evolves according to the following :

$$A_i(t+1) = (A_i(t)+1)(1-x_i(t)), t \in \{0,1,...,T\}, \forall i \in C \tag{A.1}$$

Where $x_i(t) \in \{0,1\}$ is a binary variable, representing the treatment status of cell i at time t. Correspondingly, the age vector across the landscape is $A(t) = \{A_i(t)\}_{i \in C}$.

A.2 Mature habitat and risky patch designation

Cell i is labeled 'mature' to host wildlife in year t as:

$$Mature_i\left(A(t)\right) = \begin{cases} 1 & \text{if } A_i(t) \geq m \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

Where m is the 'mature' threshold. Correspondingly, the vector of mature cells across the landscape is $Mature\left(A(t)\right) = \{Mature_i\left(A(t)\right)\}_{i \in C}$

Similarly, cell i is labeled as 'high fuel load' in year t as:

$$High_i\left(A(t)\right) = \begin{cases} 1 & \text{if } A_i(t) \geq d \\ 0 & \text{otherwise} \end{cases} \tag{A.3}$$

Where d is the 'high fuel load' threshold. Correspondingly, the vector of high fuel load cells across the landscape is $High\left(A(t)\right) = \{High_i\left(A(t)\right)\}_{i \in C}$

We assume that the maturity threshold is crossed before the high risk threshold, i.e $m < d$.

A.3 Global connectivity index and graph theory

Let a grided landscape of size n, where for each cell a_i in the set of cells A in the landscape, one defines Φ_i the set of cells connected to cell i (i.e, cells share the same status and can only be in the 8-direction direct neighborhood). Moreover, let Q_{ij} be a binary variable such that $Q_{ij} = 1$ if cells a_i and a_j are connected, 0 otherwise. Minas et al. (2014) define the following connectivity metric over a landscape:

$$\sum_{i \in C} \sum_{j \in \Phi_i} Q_{ij} \tag{A.4}$$

Now view the landscape as a graph G, with vertices V and edges E such that $G(V, E)$. For the proof, assume that Y is a binary vector such that $Y_i = 1$ if cell i is 'high risk' and 0 otherwise, and that we focus on the 'high risk' graph on the landscape. The argument is identical in the case of mature habitat.

In graph theory, an adjacency matrix $\mathcal{K}$ for an undirected graph is a binary, symmetric, square matrix of dimension $card(V)^2$ where $k_{ij} = 1$ if vertices i and j are connected, 0 otherwise. In our context, it is clear that $k_{ij} = Q_{ij}$. Equation A.4 can be reformulated as :

$$Y'\mathcal{K}Y = \sum_j \left(Y_j \sum_i Y_i k_{ij} \right) = \sum_j \left(Y_j \left(Y_j k_{jj} + \sum_{i \neq j} Y_i k_{ij} \right) \right)$$

Given the symmetric nature of $\mathcal{K}$, $\forall i \neq j$, $k_{ij} = k_{ji}$. Each cell is connected to itself so $k_{jj} = 1$.$Y_i \in \{0, 1\}$ i.e $Y_i^2 \in \{0, 1\}$:

$$\begin{aligned} Y'\mathcal{K}Y &= \sum_j \left(Y_j^2 + \sum_{i \neq j} Y_i Y_j k_{ij} \right) \\ &= \sum_j Y_j + 2 \sum_{j<i} \left(\sum_{i \neq j} Y_j Y_i a_{ij} \right) \end{aligned}$$

The first sum is the number of cells either 'mature' or 'high risk', i.e, the cardinal of the nodes of the 'high risk' graph e.g $card(V)$. In the second sum, $\sum_{i \neq j} Y_j Y_i a_{ij}$ is the number of connections of cell i to cell j, as the product $Y_i Y_j a_{ij} = 1$ if and only if cell i and j share the same status ($Y_i = Y_j$) and are in the 8-cell neighborhood ($a_{ij} = 1$). By definition, the sum of the number of connections of each cell to other cells is $card(E)$. Hence, for a set of cells C, reformulated in terms of graph theory :

$$\sum_{i \in C} \sum_{j \in \Phi_i} Q_{ij} = card(V) + 2card(E) \tag{A.5}$$

A.4 Dynamic programming equation

The Bellman equation links current and future payoffs in a recurring fashion.

$$V(t, A) = \min_{x \in \{0,1\}^{n^2}} \left(H(A) + V(t+1, A_t + 1) \right) \tag{A.6}$$

subject to constraints (2.3), (2.5), (2.4) and (2.7).

We use backward induction given by the final value $V(T, A) = H(A)$ to dynamically solve the program.

B Landscape indicators

Area We use the number of vertices (nodes) for both subgraphs and take into account cell dimensions:

$$Area(G_i) = card(V_i) \text{ for } i \in \{B, F\} \tag{B.1}$$

Number of components

$$\#components_i = card(\text{Maximal connected subgraphs of } G_i \text{ for } i \in \{B, F\}) \tag{B.2}$$

Simpson diversity index: let p_i be the proportion of land type i in the landscape. The Simpson diversity index is :

$$SIDI = 1 - \sum_i p_i^2 \tag{B.3}$$

Landscape shape index: following McGarigal and Marks (1995), the adapted LSI index from Patton (1975) in a raster landscape is:

$$LSI = \frac{0.25 \times perimeter(G)}{n} \tag{B.4}$$

Where $perimeter(G)$ is the perimeter of the cells comprised in the graph as vertices.

Land Type Heterogeneity Index: let d_{ij} be a binary variable such that $d_{ij} = 1$ if patch i and j share the same land type. Define $\mathcal{J}$ as the set of neighbors in 4 directions (north, south, east, west) of cell i[5]. The land type heterogeneity index is :

$$LTH = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\sum_{j \in \mathcal{J}_i} d_{ij}}{card(\mathcal{J}_i)} \right) \tag{B.5}$$

[5] The set $\mathcal{J}_i$ varies with cell i to account for edge effects

References

John T. Abatzoglou and A. Park Williams. Impact of anthropogenic climate change on wildfire across western us forests. *Proceedings of the National Academy of Sciences*, 113(42):11770–11775, 2016. doi: 10.1073/pnas.1607171113. URL https://www.pnas.org/doi/abs/10.1073/pnas.1607171113.

Greg J. Arthaud and Dietmar W. Rose. A methodology for estimating production possibility frontiers for wildlife habitat and timber value at the landscape level. *Canadian Journal of Forest Research*, 26(12):2191–2200, December 1996. ISSN 0045-5067. doi: 10.1139/x26-248. URL https://cdnsciencepub.com/doi/10.1139/x26-248. Publisher: NRC Research Press.

Ashwin Arulselvan, Clayton W. Commander, Lily Elefteriadou, and Panos M. Pardalos. Detecting critical nodes in sparse graphs. *Computers Operations Research*, 36(7):2193–2200, 2009. ISSN 0305-0548. doi: https://doi.org/10.1016/j.cor.2008.08.016. URL https://www.sciencedirect.com/science/article/pii/S0305054808001494.

Richard Bellman. *Dynamic Programming*. Princeton University Press, 1957.

Matthias M. Boer, Rohan J. Sadler, Roy S. Wittkuhn, Lachlan McCaw, and Pauline F. Grierson. Long-term impacts of prescribed burning on regional extent and incidence of wildfires—Evidence from 50 years of active fire management in SW Australian forests. *Forest Ecology and Management*, 259(1):132–142, 2009. ISSN 0378-1127. doi: https://doi.org/10.1016/j.foreco.2009.10.005. URL https://www.sciencedirect.com/science/article/pii/S0378112709007294.

S. D. Bradshaw, K.W. Dixon, H. Lambers, A.T. Cross, J. Bailey, and S.D. Hopper. Understanding the long-term impact of prescribed burning in mediterranean-climate biodiversity hotspots, with a focus on south-western australia. *International Journal of Wildland Fire*, 27, 2018.

R. A. Bradstock, M. Bedward, A. M. Gill, J. S. Cohn, R. A. Bradstock, M. Bedward, A. M. Gill, and J. S. Cohn. Which mosaic? A landscape ecological approach for evaluating interactions between fire regimes, habitat and animals. *Wildlife Research*, 32(5): 409–423, August 2005. ISSN 1448-5494, 1448-5494. doi: 10.1071/WR02114. URL https://www.publish.csiro.au/wr/WR02114.

Marshall Burke, Marissa L. Childs, Brandon De la Cuesta, Minghao Qiu, Jessica Li, Carlos F. Gould, Sam Heft-Neal, and Michael Wara. Wildfire Influence on Recent US Pollution Trends, January 2023. URL https://www.nber.org/papers/w30882.

Neil Burrows and Lachlan McCaw. Prescribed burning in southwestern australian forests. *Frontiers in Ecology and the Environment*, 11 (s1):e25–e34, 2013. doi: https://doi.org/10.1890/120356. URL https://esajournals.onlinelibrary.wiley.com/doi/abs/10.1890/120356.

David E Calkin, Susan Stevens Hummel, and James K Agee. Modeling trade-offs between fire threat reduction and late-seral forest structure. *Canadian Journal of Forest Research*, 35(11):2562–2574, November 2005. ISSN 0045-5067. doi: 10.1139/x05-177. URL https://cdnsciencepub.com/doi/10.1139/x05-177. Publisher: NRC Research Press.

Jonathan D Coop, Sean A Parks, Camille S Stevens-Rumann, Shelley D Crausbay, Philip E Higuera, Matthew D Hurteau, Alan Tepley, Ellen Whitman, Timothy Assal, Brandon M Collins, Kimberley T Davis, Solomon Dobrowski, Donald A Falk, Paula J Fornwalt, Peter Z Fulé, Brian J Harvey, Van R Kane, Caitlin E Littlefield, Ellis Q Margolis, Malcolm North, Marc-André Parisien, Susan Prichard, and Kyle C Rodman. Wildfire-Driven Forest Conversion in Western North American Landscapes. *BioScience*, 70(8): 659–673, August 2020. ISSN 0006-3568. doi: 10.1093/biosci/biaa061. URL https://doi.org/10.1093/biosci/biaa061.

Lenore Fahrig, Jacques Baudry, Lluís Brotons, Françoise G. Burel, Thomas O. Crist, Robert J. Fuller, Clelia Sirami, Gavin M. Siriwardena, and Jean-Louis Martin. Functional landscape heterogeneity and animal biodiversity in agricultural landscapes. *Ecology Letters*, 14(2):101–112, 2011. doi: https://doi-org.inshs.bib.cnrs.fr/10.1111/j.1461-0248.2010.01559.x. URL https://onlinelibrary-wiley-com.inshs.bib.cnrs.fr/doi/abs/10.1111/j.1461-0248.2010.01559.x.

Paulo M Fernandes, G Matt Davies, Davide Ascoli, Cristina Fernández, Francisco Moreira, Eric Rigolot, Cathelijne R Stoof, José Antonio Vega, and Domingo Molina. Prescribed burning in southern europe: developing fire management in a dynamic landscape. *Frontiers in Ecology and the Environment*, 11(s1):e4–e14, 2013. doi: https://doi.org/10.1890/120298. URL https://esajournals.onlinelibrary.wiley.com/doi/abs/10.1890/120298.

Mark A. Finney. Design of Regular Landscape Fuel Treatment Patterns for Modifying Fire Growth and Behavior. *Forest Science*, 47 (2):219–228, May 2001. ISSN 0015-749X. doi: 10.1093/forestscience/47.2.219. URL https://doi.org/10.1093/forestscience/47.2.219.

Mark A. Finney. A computational method for optimising fuel treatment locations. *International Journal of Wildland Fire*, 16(6): 702–711, December 2007. ISSN 1448-5516, 1448-5516. doi: 10.1071/WF06063. URL https://www.publish.csiro.au/wf/WF06063. Publisher: CSIRO PUBLISHING.

Veronique Florec, Michael Burton, David Pannell, Joel Kelso, and George Milne. Where to prescribe burn: the costs and benefits of prescribed burning close to houses. *International Journal of Wildland Fire*, (29), 2020.

Jacob Gellman, Margaret Walls, and Matthew Wibbenmeyer. Welfare losses from wildfire smoke: evidence from daily outdoor recreation data. *Resources for the Future Working Papers*, 2023.

Jose Ramon Gonzalez-Olabarria, Jaime Carrasco, Cristobal Pais, Jordi Garcia-Gonzalo, David Palacios-Meneses, Rodrigo Mahaluf-Recasens, Olena Porkhum, and Andrés Weintraub. A fire spread simulator to support tactical management decisions for Mediterranean landscapes. *Frontiers in Forests and Global Change*, 6, 2023. ISSN 2624-893X. URL https://www.frontiersin.org/articles/10.3389/ffgc.2023.1071484.

Sam Heft-Neal, Carlos F. Gould, Marissa Childs, Mathew V. Kiang, Kari Nadeau, Mark Duggan, Eran Bendavid, and Marshall Burke. Behavior Mediates the Health Effects of Extreme Wildfire Smoke Events, February 2023. URL https://www.nber.org/papers/w30969.

W. Matt Jolly, Mark A. Cochrane, Patrick H. Freeborn, Zachary A. Holden, Timothy J. Brown, Grant J. Williamson, and David M. J. S. Bowman. Climate-induced variations in global wildfire danger from 1979 to 2013. *Nature Communications*, 6(1):7537, July 2015. ISSN 2041-1723. doi: 10.1038/ncomms8537. URL https://doi.org/10.1038/ncomms8537.

Elizabeth L. Kalies and Larissa L. Yocom Kent. Tamm review: Are fuel treatments effective at achieving ecological and social objectives? a systematic review. *Forest Ecology and Management*, 375:84–95, 2016. ISSN 0378-1127. doi: https://doi.org/10.1016/j.foreco.2016.05.021. URL https://www.sciencedirect.com/science/article/pii/S0378112716302626.

Masashi Konoshima, Claire A. Montgomery, Heidi J. Albers, and Jeffrey L. Arthur. Spatial-Endogenous Fire Risk and Efficient Fuel Management and Timber Harvest. *Land Economics*, 84(3):449–468, August 2008. ISSN 0023-7639, 1543-8325. doi: 10.3368/le.84.3.449. URL https://le.uwpress.org/content/84/3/449. Publisher: University of Wisconsin Press Section: Articles.

J.A Kupfer, A. Terando, P. Gao, C. Teske, and J.K. Hiers. Climate change projected to reduce prescribed burning opportunities in the south-eastern united states. *International Journal of Wildland Fire*, 29, 2020.

John F. Lehmkuhl, Maureen Kennedy, E. David Ford, Peter H. Singleton, William L. Gaines, and Rick L. Lind. Seeing the forest for the fuel: Integrating ecological values and fuels management. *Forest Ecology and Management*, 246(1):73–80, 2007. ISSN 0378-1127. doi: https://doi.org/10.1016/j.foreco.2007.03.071. URL https://www.sciencedirect.com/science/article/pii/S0378112707002800.

Damon B. Lesmeister, Raymond J. Davis, Stan G. Sovern, and Zhiqiang Yang. Northern spotted owl nesting forests as fire refugia: a 30-year synthesis of large wildfires. *Fire Ecology*, 17(1):32, November 2021. ISSN 1933-9747. doi: 10.1186/s42408-021-00118-z. URL https://doi.org/10.1186/s42408-021-00118-z.

Susan C. Loeb and Rachel V. Blakey. Bats and fire: a global review. *Fire Ecology*, 17(1):29, November 2021. ISSN 1933-9747. doi: 10.1186/s42408-021-00109-0. URL https://doi.org/10.1186/s42408-021-00109-0.

Kathryn E. Low, John J. Battles, Ryan E. Tompkins, Colin P. Dillingham, Scott L. Stephens, and Brandon M. Collins. Shaded fuel breaks create wildfire-resilient forest stands: lessons from a long-term study in the Sierra Nevada. *Fire Ecology*, 19(1):29, May 2023. ISSN 1933-9747. doi: 10.1186/s42408-023-00187-2. URL https://doi.org/10.1186/s42408-023-00187-2.

Dmytro Matsypura, Oleg A. Prokopyev, and Aizat Zahar. Wildfire fuel management: Network-based models and optimization of prescribed burning. *European Journal of Operational Research*, 264(2):774–796, January 2018. ISSN 03772217. doi: 10.1016/j.ejor.2017.06.050. URL https://linkinghub.elsevier.com/retrieve/pii/S037722171730591X.

Michael A. McCarthy, A. Malcolm Gill, and Ross A. Bradstock. Theoretical fire-interval distributions. *International Journal of Wildland Fire*, 10(1):73–77, 2001. URL https://doi.org/10.1071/WF01013.

Kevin McGarigal and Barbara J. Marks. FRAGSTATS: spatial pattern analysis program for quantifying landscape structure. Technical report, 1995. URL https://doi.org/10.2737%2Fpnw-gtr-351.

James Minas, John W. Hearne, and David L. Martell. A spatial optimisation model for multi-period landscape level fuel management to mitigate wildfire impacts. *European Journal of Operational Research*, 232(2):412–422, January 2014. ISSN 0377-2217. doi: 10.1016/j.ejor.2013.07.026. URL https://www.sciencedirect.com/science/article/pii/S0377221713006073. Publisher: North-Holland.

Emily S. Minor and Dean L. Urban. A Graph-Theory Framework for Evaluating Landscape Connectivity and Conservation Planning. *Conservation Biology*, 22(2):297–307, 2008. ISSN 1523-1739. doi: 10.1111/j.1523-1739.2007.00871.x. URL http://onlinelibrary.wiley.com/doi/abs/10.1111/j.1523-1739.2007.00871.x. _eprint: https://conbio.onlinelibrary.wiley.com/doi/pdf/10.1111/j.1523-1739.2007.00871.x.

L. Mouysset, L. Doyen, and F. Jiguet. How does economic risk aversion affect biodiversity? *Ecological Applications*, 23(1):96–109, 2013. ISSN 10510761. URL http://www.jstor.org/stable/23440820.

United States Government Accountability Office. Wildland fire - federal agencies' efforts to reduce wildland fuels and lower risk to communities and ecosystems. Technical report, United States Government Accountability Office, 2019.

Andrew D. Olds, Kylie A. Pitt, Paul S. Maxwell, and Rod M. Connolly. Synergistic effects of reserves and connectivity on ecological resilience. *Journal of Applied Ecology*, 49(6):1195–1203, 2012. doi: https://doi.org/10.1111/jpe.12002. URL https://besjournals.onlinelibrary.wiley.com/doi/abs/10.1111/jpe.12002.

Jerry S. Olson. Energy storage and the balance of producers and decomposers in ecological systems. *Ecology*, 44(2):322–331, 1963. doi: https://doi-org.inshs.bib.cnrs.fr/10.2307/1932179. URL https://esajournals-onlinelibrary-wiley-com.inshs.bib.cnrs.fr/doi/abs/10.2307/1932179.

Cristobal Pais, Jaime Carrasco, Pelagie Elimbi Moudio, and Zuo-Jun Max Shen. Downstream protection value: Detecting critical zones for effective fuel-treatment under wildfire risk. *Computers & Operations Research*, 131, July 2021a. ISSN 0305-0548. doi: 10.1016/j.cor.2021.105252. URL https://www.sciencedirect.com/science/article/pii/S0305054821000447.

Cristobal Pais, Jaime Carrasco, David L. Martell, Andres Weintraub, and David L. Woodruff. Cell2Fire: A Cell-Based Forest Fire Growth Model to Support Strategic Landscape Management Planning. *Frontiers in Forests and Global Change*, 4, 2021b. ISSN 2624-893X. URL https://www.frontiersin.org/articles/10.3389/ffgc.2021.692706.

Lucía Pascual-Hortal and Santiago Saura. Comparison and development of new graph-based landscape connectivity indices: towards the priorization of habitat patches and corridors for conservation. *Landscape Ecology*, 21(7):959–967, October 2006. ISSN 0921-2973, 1572-9761. doi: 10.1007/s10980-006-0013-z. URL http://link.springer.com/10.1007/s10980-006-0013-z.

David R. Patton. A Diversity Index for Quantifying Habitat "Edge". *Wildlife Society Bulletin (1973-2006)*, 3(4):171–173, 1975. ISSN 0091-7648. URL https://www.jstor.org/stable/3781151. Publisher: [Wiley, Wildlife Society].

Seth H. Peterson, Marco E. Morais, Jean M. Carlson, Philip E. Dennison, Dar A. Roberts, Max A. Moritz, and David R. Weise. Using HFire for spatial modeling of fire in shrublands. Technical report, 2009. URL https://doi.org/10.2737%2Fpsw-rp-259.

Ramya Rachmawati, Melih Ozlen, Karin J. Reinke, and John W. Hearne. A model for solving the prescribed burn planning problem. *SpringerPlus*, 4(1):630, October 2015. ISSN 2193-1801. doi: 10.1186/s40064-015-1418-4. URL https://doi.org/10.1186/s40064-015-1418-4.

Ramya Rachmawati, Melih Ozlen, Karin J. Reinke, and John W. Hearne. An optimisation approach for fuel treatment planning to break the connectivity of high-risk regions. *Forest Ecology and Management*, 368:94–104, May 2016. ISSN 0378-1127. doi: 10.1016/j.foreco.2016.03.014. URL https://www.sciencedirect.com/science/article/pii/S0378112716300731.

Ramya Rachmawati, Melih Ozlen, John Hearne, and Karin Reinke. Fuel treatment planning: Fragmenting high fuel load areas while maintaining availability and connectivity of faunal habitat. *Applied Mathematical Modelling*, 54:298–310, February 2018. ISSN 0307-904X. doi: 10.1016/j.apm.2017.09.045. URL https://www.sciencedirect.com/science/article/pii/S0307904X17306066.

Bronwyn Rayfield, David Pelletier, Maria Dumitru, Jeffrey A. Cardille, and Andrew Gonzalez. Multipurpose habitat networks for short-range and long-range connectivity: a new method combining graph and circuit connectivity. *Methods in Ecology and Evolution*, 7(2): 222–231, February 2016. ISSN 2041-210X. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/2041-210X.12470. Publisher: John Wiley & Sons, Ltd.

Julien Ruffault, Thomas Curt, Nicolas K. Martin-StPaul, Vincent Moron, and Ricardo M. Trigo. Extreme wildfire events are linked to global-change-type droughts in the northern Mediterranean. *Natural Hazards and Earth System Sciences*, 18(3):847–856, March 2018. ISSN 1561-8633. doi: 10.5194/nhess-18-847-2018. URL https://nhess.copernicus.org/articles/18/847/2018/. Publisher: Copernicus GmbH.

Adam Rytwinski and Kevin A. Crowe. A simulation-optimization model for selecting the location of fuel-breaks to minimize expected losses from forest fires. *Forest Ecology and Management*, 260(1):1–11, June 2010. ISSN 0378-1127. doi: 10.1016/j.foreco.2010.03.013. URL https://www.sciencedirect.com/science/article/pii/S0378112710001593.

Victoria A. Saab, Quresh R. Latif, William M. Block, and Jonathan G. Dudley. Short-term benefits of prescribed fire to bird communities of dry forests. *Fire Ecology*, 18(1):4, April 2022. ISSN 1933-9747. doi: 10.1186/s42408-022-00130-x. URL https://doi.org/10.1186/s42408-022-00130-x.

C. E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, 1948. doi: https://doi.org/10.1002/j.1538-7305.1948.tb01338.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/j.1538-7305.1948.tb01338.x.

E. H. Simpson. Measurement of Diversity. *Nature*, 163(4148):688–688, April 1949. ISSN 1476-4687. doi: 10.1038/163688a0. URL https://doi.org/10.1038/163688a0.

Holly Sitters, Julian Di Stefano, Fiona J. Christie, Paul Sunnucks, and Alan York. Bird diversity increases after patchy prescribed fire: implications from a before-after control-impact study. *International Journal of Wildland Fire*, 24:690–701, 2015. URL https://api.semanticscholar.org/CorpusID:129053082.

Kristin Sweeney, Ruth Dittrich, Spencer Moffat, Chelsea Power, and Jeffrey D. Kline. Estimating the economic value of carbon losses from wildfires using publicly available data sources: Eagle Creek Fire, Oregon 2017. *Fire Ecology*, 19(1):55, September 2023. ISSN 1933-9747. doi: 10.1186/s42408-023-00206-2. URL https://doi.org/10.1186/s42408-023-00206-2.

Chris Taylor, Michael A. McCarty, and David B. Lindemayer. Nonlinear effects of stand age on fire severity. *Conservation Letters*, (7), 2014. doi: doi:10.1111/conl.12122.

Philip D. Taylor, Lenore Fahrig, Kringen Henein, and Gray Merriam. Connectivity is a vital element of landscape structure. *Oikos*, 68 (3):571–573, 1993. ISSN 00301299, 16000706. URL http://www.jstor.org/stable/3544927.

Alan R Templeton, Hilary Brazeal, and Jennifer L Neuwald. The transition from isolated patches to a metapopulation in the eastern collared lizard in response to prescribed fires. *Ecology*, 92(9):1736–1747, 2011. doi: https://doi.org/10.1890/10-1994.1. URL https://esajournals.onlinelibrary.wiley.com/doi/abs/10.1890/10-1994.1.

Yuhong Tian, Yiqing Liu, C. Y. Jim, and Hanzhang Song. Assessing Structural Connectivity of Urban Green Spaces in Metropolitan Hong Kong. *Sustainability*, 9(9):1653, September 2017. ISSN 2071-1050. doi: 10.3390/su9091653. URL https://www.mdpi.com/2071-1050/9/9/1653. Number: 9.

Monica G. Turner. Landscape ecology: What is the state of the science? *Annual Review of Ecology, Evolution, and Systematics*, 36 (1):319–344, 2005. doi: 10.1146/annurev.ecolsys.36.102003.152614. URL https://doi.org/10.1146/annurev.ecolsys.36.102003.152614.

Monica G. Turner and Robert H. Gardner. *Landscape Ecology in Theory and Practice*. Springer New York, 2015.

Dean Urban and Timothy Keitt. Landscape Connectivity: A Graph-Theoretic Perspective. *Ecology*, 82(5):1205–1218, 2001. ISSN 1939-9170. doi: 10.1890/0012-9658(2001)082[1205:LCAGTP]2.0.CO;2. URL https://onlinelibrary.wiley.com/doi/abs/10.1890/0012-9658%282001%29082%5B1205%3ALCAGTP%5D2.0.CO%3B2.

Nicole Vaillant, Jo Ann Fites-Kaufman, and Scott L. Stephens. Effectiveness of prescribed fire as a fuel treatment in californian coniferous forests. *International Journal of Wildland Fire*, (18), 2009.

J. Velázquez, J. Gutiérrez, A. García-Abril, A. Hernando, M. Aparicio, and B. Sánchez. Structural connectivity as an indicator of species richness and landscape diversity in Castilla y León (Spain). *Forest Ecology and Management*, 432:286–297, 2019. ISSN 0378-1127. doi: https://doi.org/10.1016/j.foreco.2018.09.035. URL https://www.sciencedirect.com/science/article/pii/S0378112718312957.

Daoping Wang, Dabo Guan, Shupeng Zhu, Michael Mac Kinnon, Guannan Geng, Qiang Zhang, Heran Zheng, Tianyang Lei, Shuai Shao, Peng Gong, and Steven J. Davis. Economic footprint of California wildfires in 2018. *Nature Sustainability*, 4(3):252–260, March 2021. ISSN 2398-9629. doi: 10.1038/s41893-020-00646-7. URL https://doi.org/10.1038/s41893-020-00646-7.

Tzeidle N. Wasserman and Stephanie E. Mueller. Climate influences on future fire severity: a synthesis of climate-fire interactions and impacts on fire regimes, high-severity fire, and forests in the western United States. *Fire Ecology*, 19(1):43, July 2023. ISSN 1933-9747. doi: 10.1186/s42408-023-00200-8. URL https://doi.org/10.1186/s42408-023-00200-8.

Yu Wei, Douglas Rideout, and Andy Kirsch. An optimization model for locating fuel treatments across a landscape to reduce expected fire losses. *Canadian Journal of Forest Research*, 38(4):868–877, April 2008. ISSN 0045-5067. doi: 10.1139/X07-162. URL https://cdnsciencepub.com/doi/10.1139/X07-162. Publisher: NRC Research Press.

Brooke A. Williams, Luke P. Shoo, Kerrie A. Wilson, and Hawthorne L. Beyer. Optimising the spatial planning of prescribed burns to achieve multiple objectives in a fire-dependent ecosystem. *Journal of Applied Ecology*, 54(6):1699–1709, 2017. doi: https://doi.org/10.1111/1365-2664.12920. URL https://besjournals.onlinelibrary.wiley.com/doi/abs/10.1111/1365-2664.12920.

Brendan A. Wintle, Sarah Legge, and John C.Z. Woinarski. After the megafires: What next for australian wildlife? *Trends in Ecology Evolution*, 35(9):753–757, 2020. ISSN 0169-5347. doi: https://doi.org/10.1016/j.tree.2020.06.009. URL https://www.sciencedirect.com/science/article/pii/S0169534720301713.

Denys Yemshanov, Ning Liu, Daniel K. Thompson, Marc-André Parisien, Quinn E. Barber, Frank H. Koch, and Jonathan Reimer. Detecting critical nodes in forest landscape networks to reduce wildfire spread. *PLOS ONE*, 16(10):e0258060, October 2021. ISSN 1932-6203. doi: 10.1371/journal.pone.0258060. URL https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0258060. Publisher: Public Library of Science.

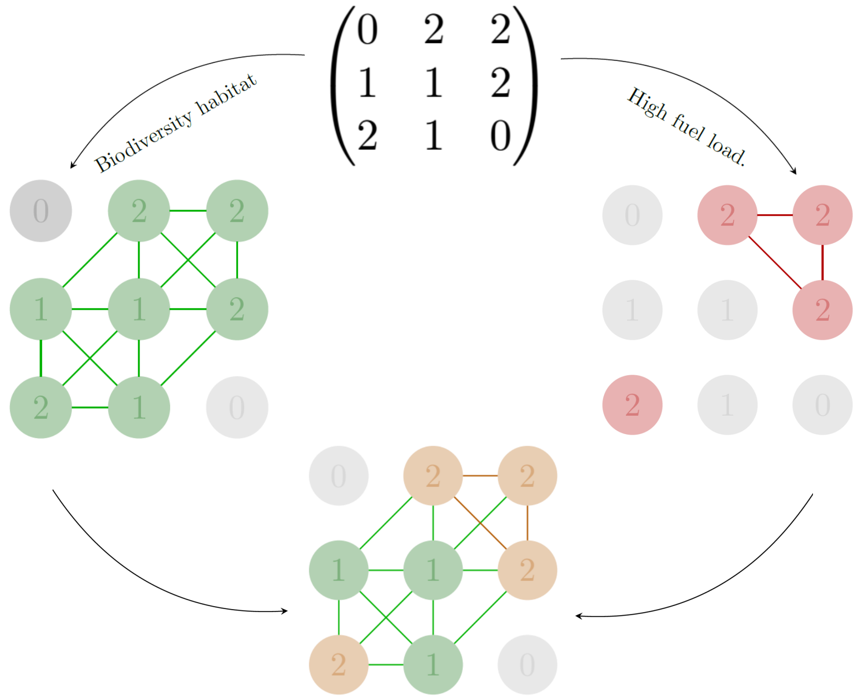
Denys Yemshanov, Robert G. Haight, Ning Liu, Rob Rempel, Frank H. Koch, and Art Rodgers. Exploring the tradeoffs among forest planning, roads and wildlife corridors: a new approach. *Optimization Letters*, 16(3):747–788, April 2022. ISSN 1862-4472, 1862-4480. doi: 10.1007/s11590-021-01745-w. URL https://link.springer.com/10.1007/s11590-021-01745-w.

Bo Zheng, Philippe Ciais, Frederic Chevallier, Hui Yang, Josep G. Canadell, Yang Chen, Ivar R. van der Velde, Ilse Aben, Emilio Chuvieco, Steven J. Davis, Merritt Deeter, Chaopeng Hong, Yawen Kong, Haiyan Li, Hui Li, Xin Lin, Kebin He, and Qiang Zhang. Record-high CO2 emissions from boreal fires in 2021. *Science*, 379(6635):912–917, March 2023. doi: 10.1126/science.ade0805. URL https://www.science.org/doi/10.1126/science.ade0805. Publisher: American Association for the Advancement of Science.

Yangming Zhou and Jin-Kao Hao. A fast heuristic algorithm for the critical node problem. In *Proceedings of the Genetic and Evolutionary Computation Conference Companion*, GECCO '17, page 121–122, New York, NY, USA, 2017. Association for Computing Machinery. ISBN 9781450349390. doi: 10.1145/3067695.3075993. URL https://doi.org/10.1145/3067695.3075993.

A Figures

Figure 1: Illustration of the habitat and fuel graphs for $n = 3$



In this graph, green cells support biodiversity habitat only, while red cells display high risk.

The high risk graph has two components (top right corner with 3 nodes, and bottom left corner with 1 node), while the biodiversity habitat graph only has one.

Cells for which the value is 0 are not considered as nodes for both graphs, and are thus not connected to the rest of the graphs. In the end, because high fuel load cells also support biodiversity habitat, the landscape can be represented as the overlap between the two graphs, where orange cells are high fuel load and also support biodiversity habitat.

Figure 2: Number of cycles as a function of biodiversity habitat and budget

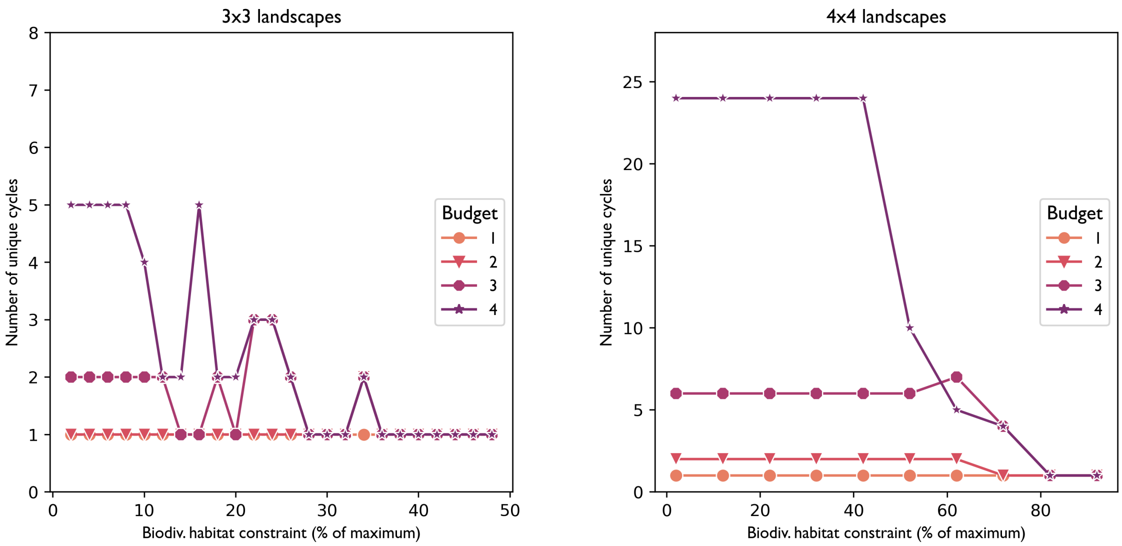


Figure 3: Production possibility frontier between constraint (as a % of maximum biodiversity sustainable in landscape) and wildfire risk for various budgets, and landscape size

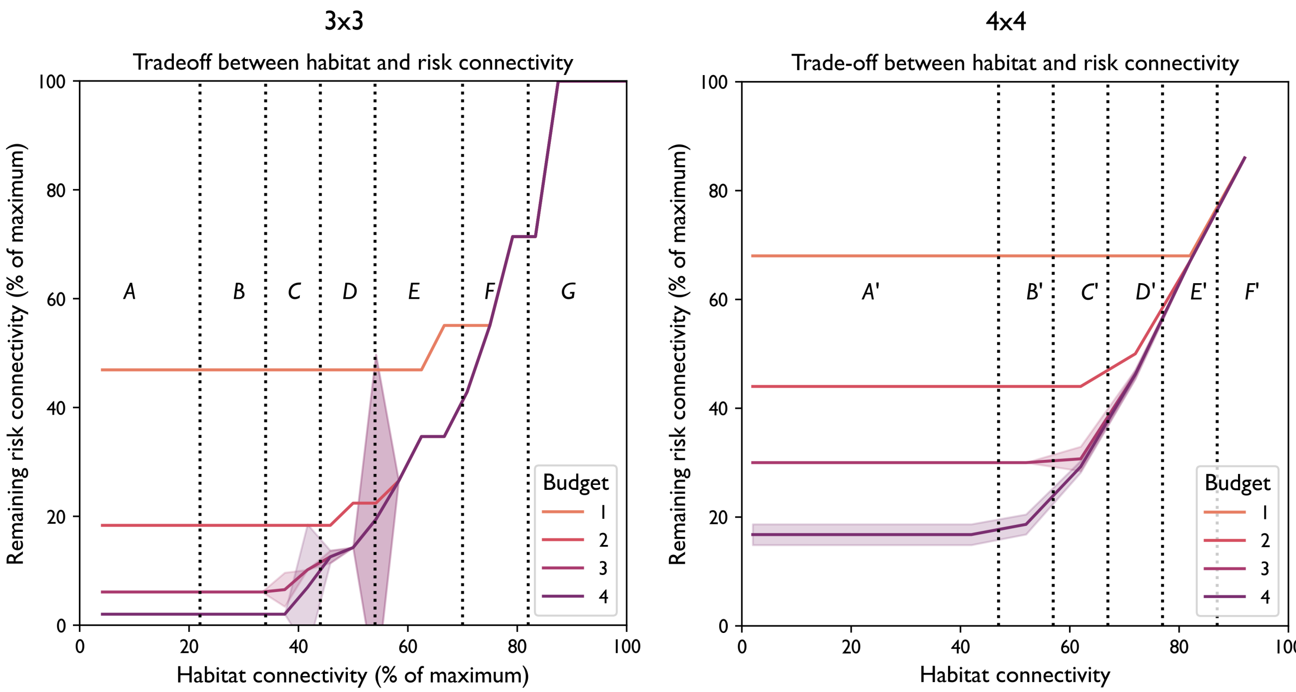


Figure 4: Most represented cycles for each biodiversity constraint level, for various budget and landscapes 3×3, and 4×4 (95% CI shaded)

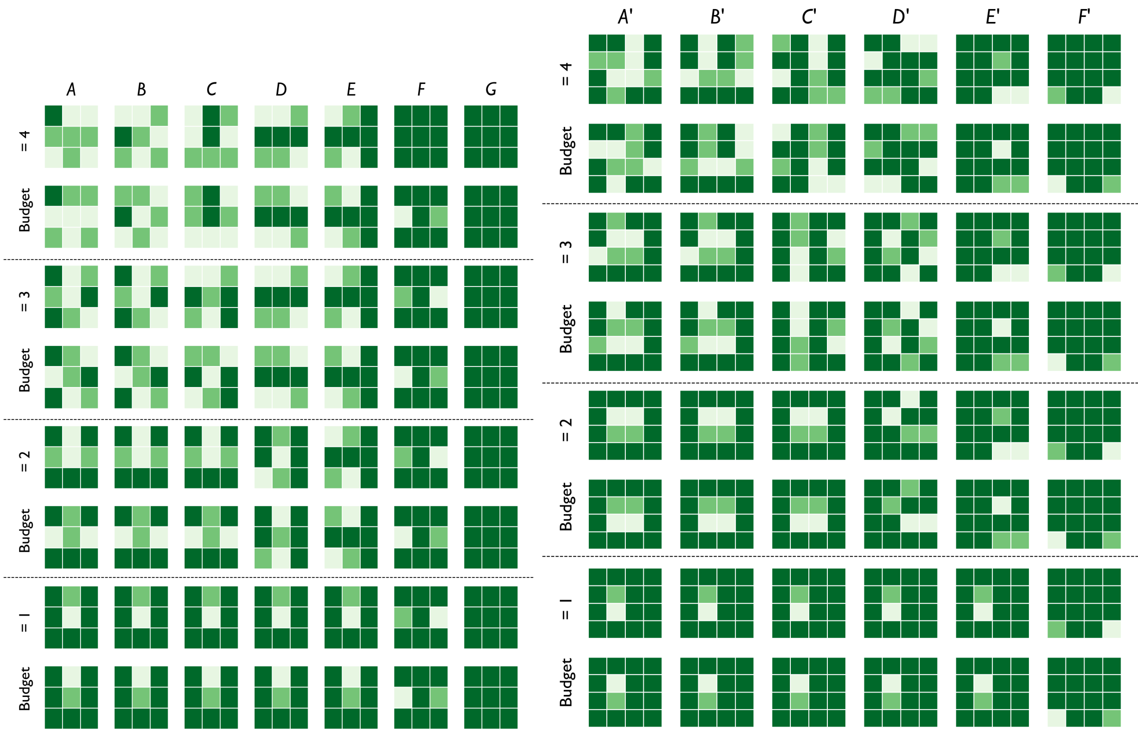


Figure 5: Assessment: surface, components of high-risk graph (95% CI shaded)

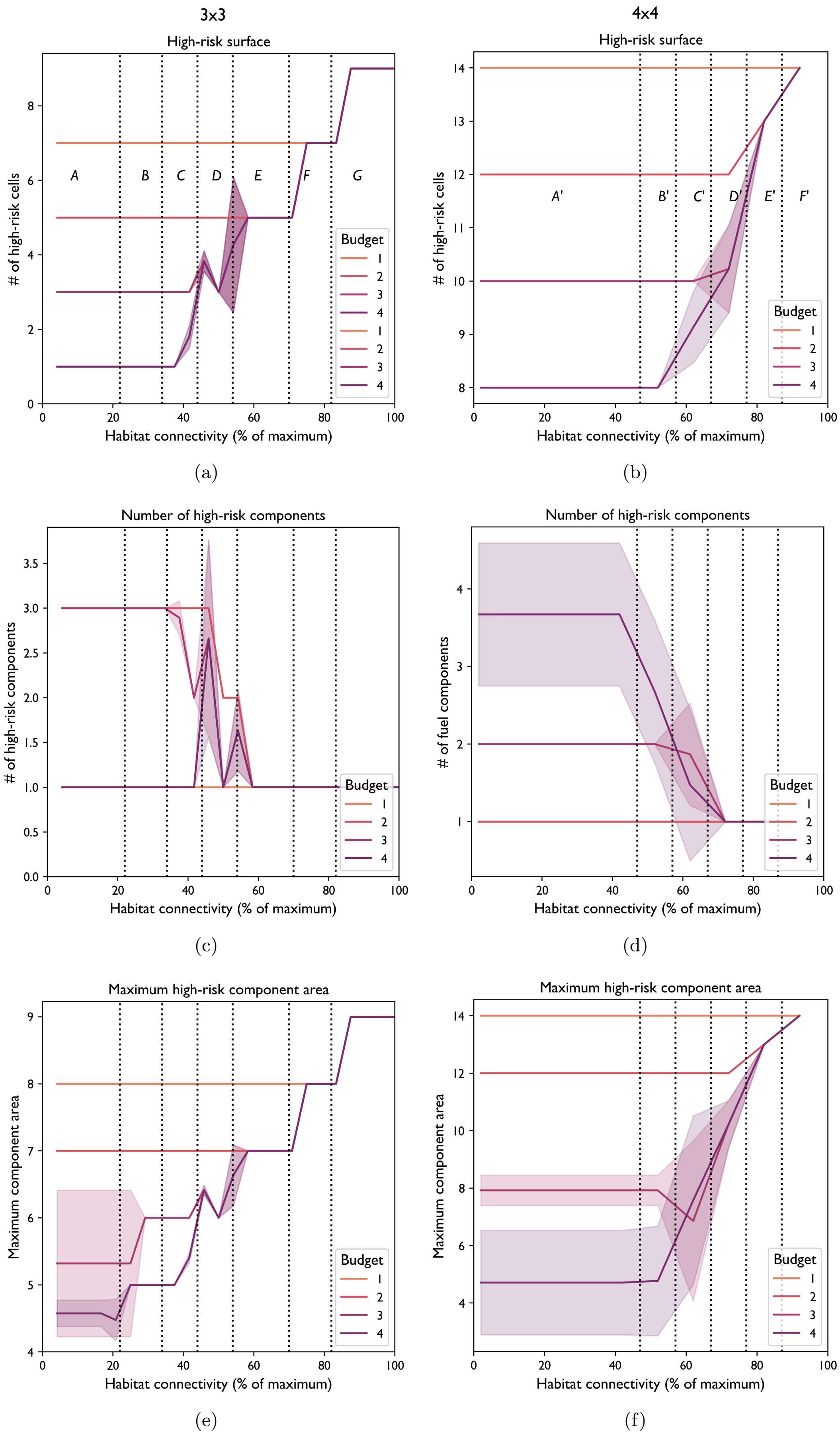
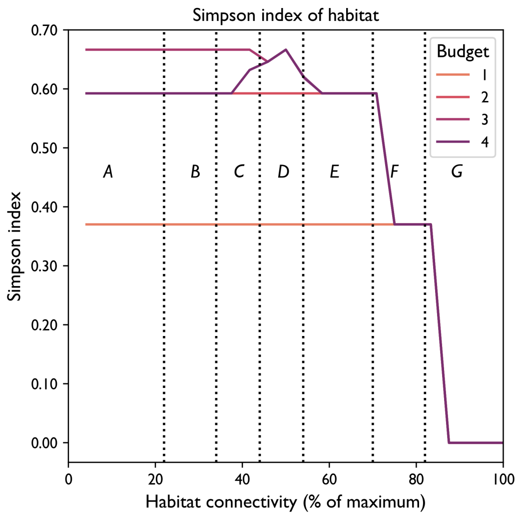
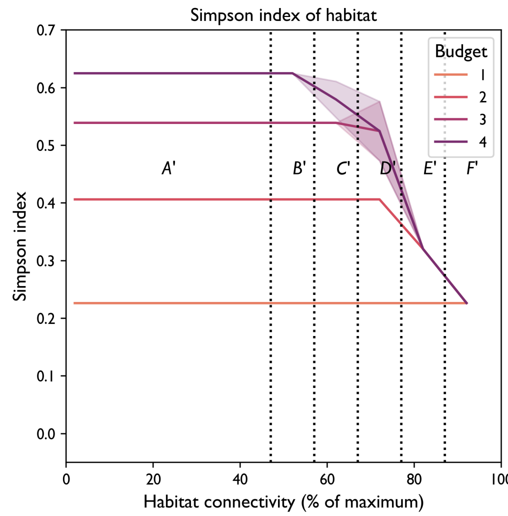


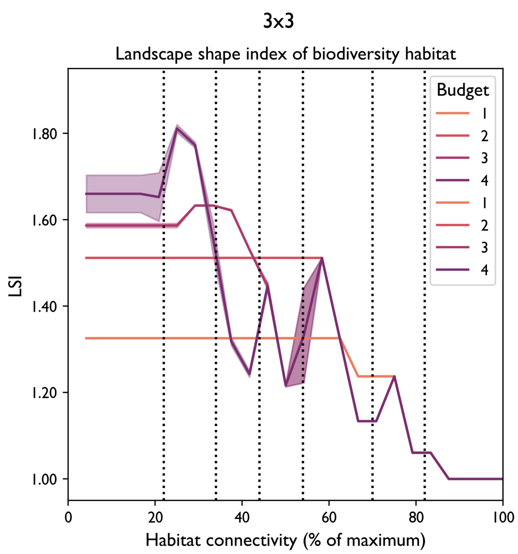
Figure 6: Assessment: diversity (95% CI shaded)



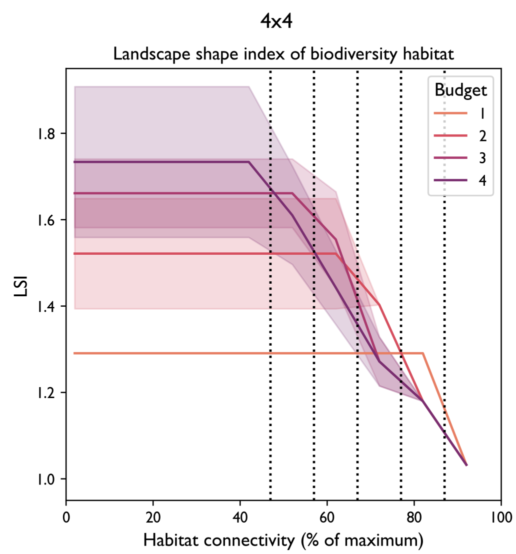
(a)



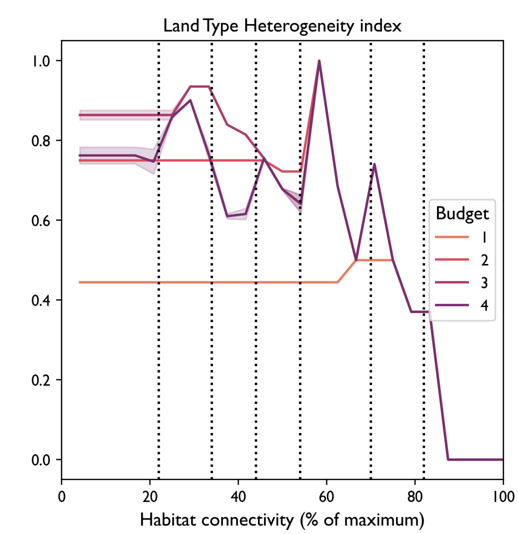
(b)



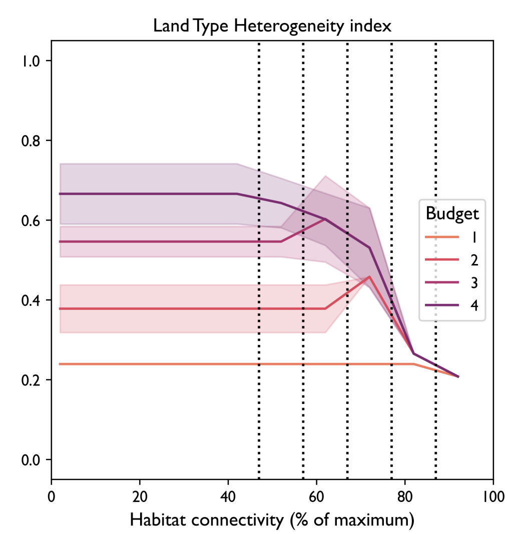
(c)



(d)

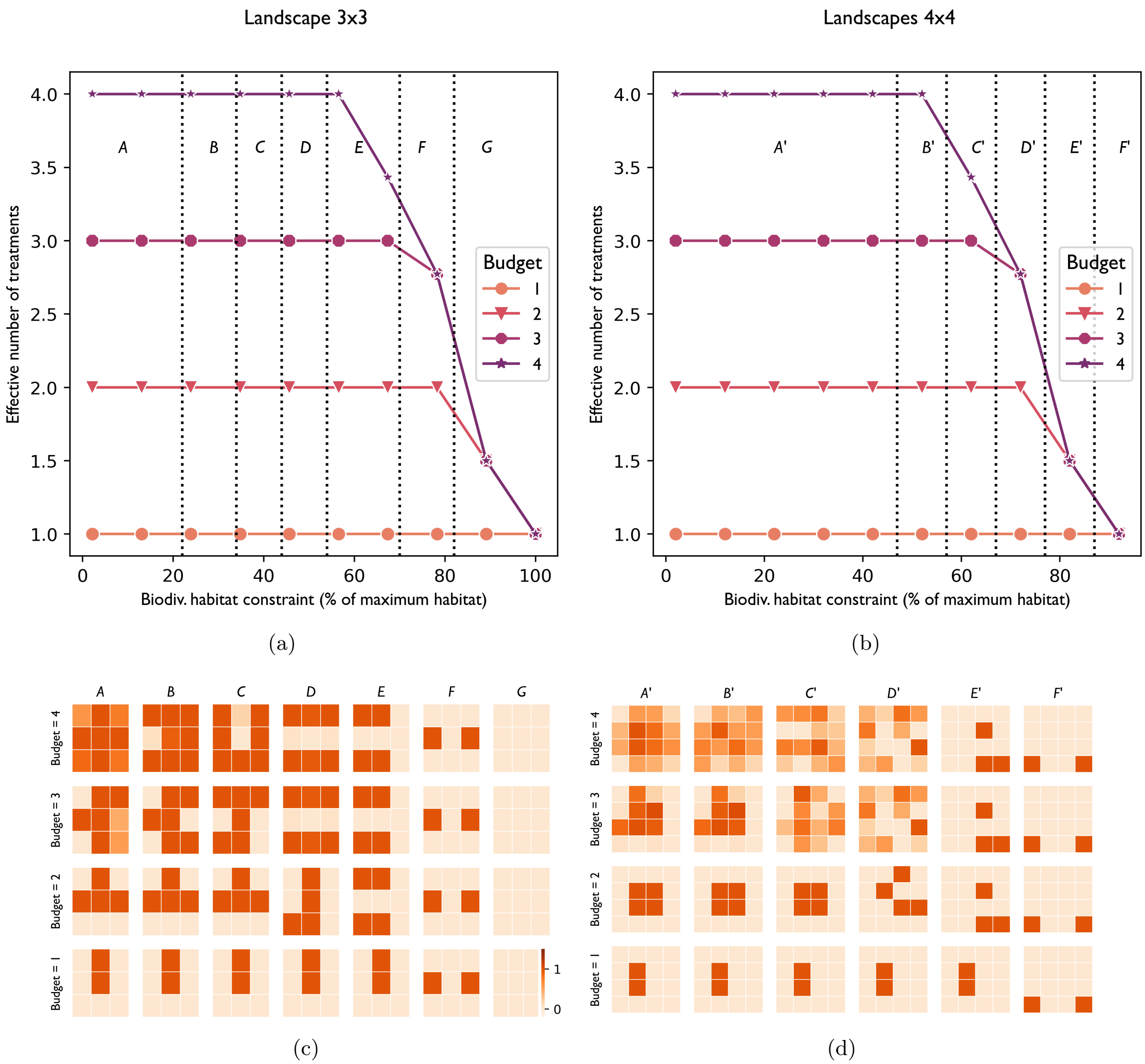


(e)



(f)

Figure 7: Treatment allocation : number, location



(a) (b)

(c) (d)